DYALOG

Belfast 2018

How many dimensions does a vector have?

Nicolas Delcros

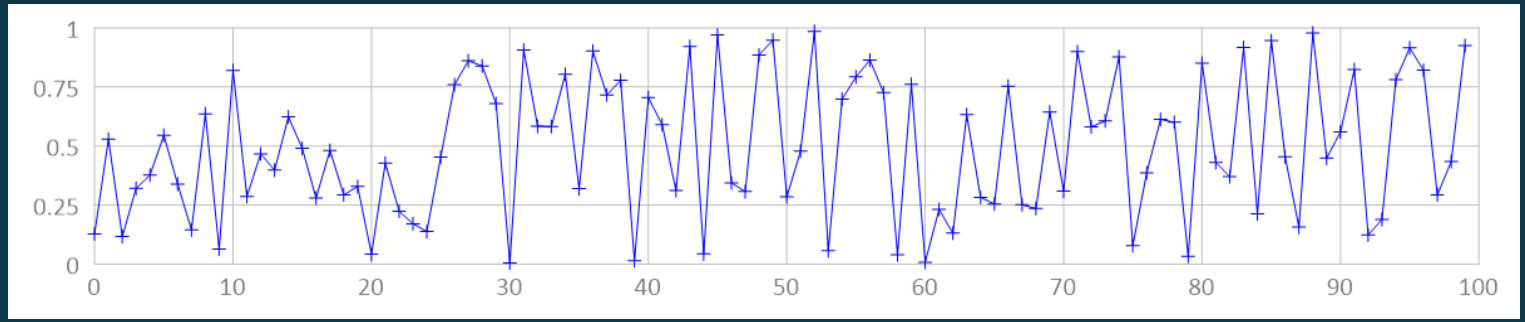
How many dimensions does a vector have?

- Math: N scalars
- APL: 1 axis
- Chart: 2 degrees of freedom
- Geek: real or ~~complex~~?

- We're interested in degrees of freedom of a real vector

How many dimensions does a vector have?

- `vec←?100⍴0`



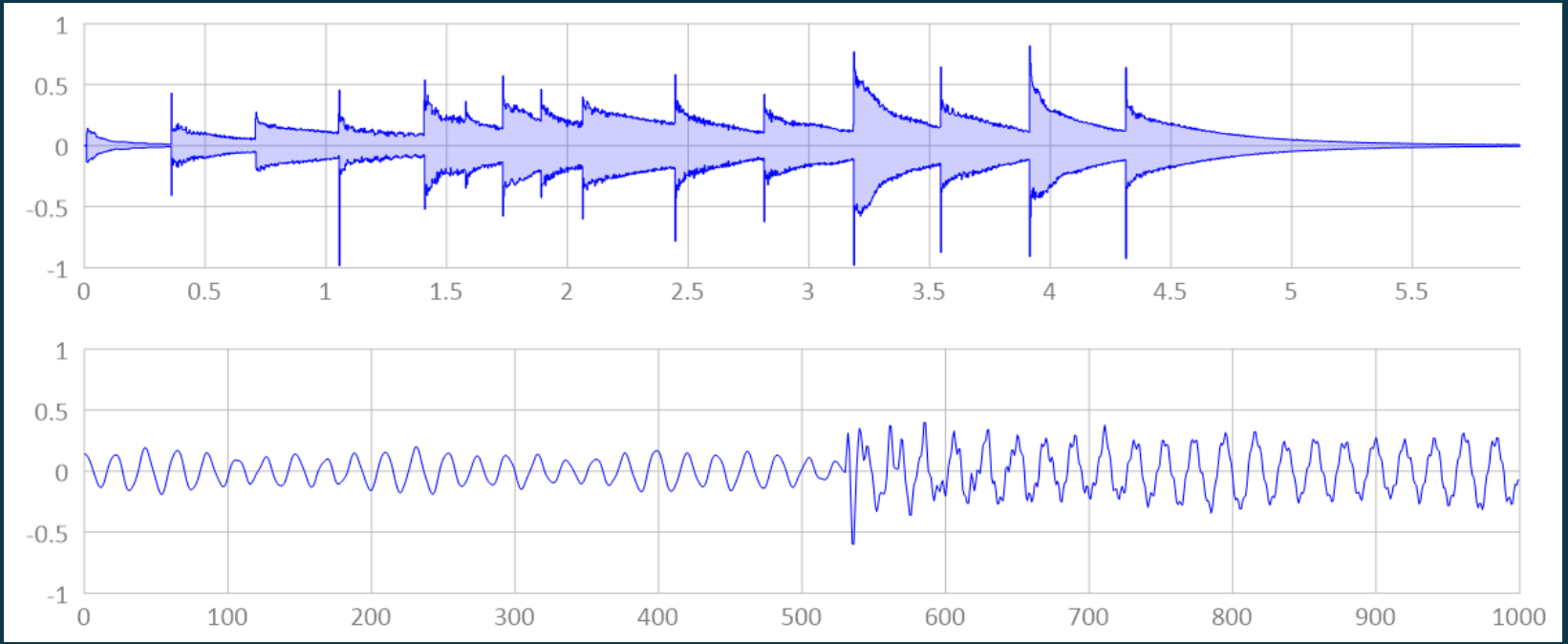
- Domain: 1d (e.g. time)
- Image: 1d (e.g. acoustic pressure)

Degrees of freedom of a sound

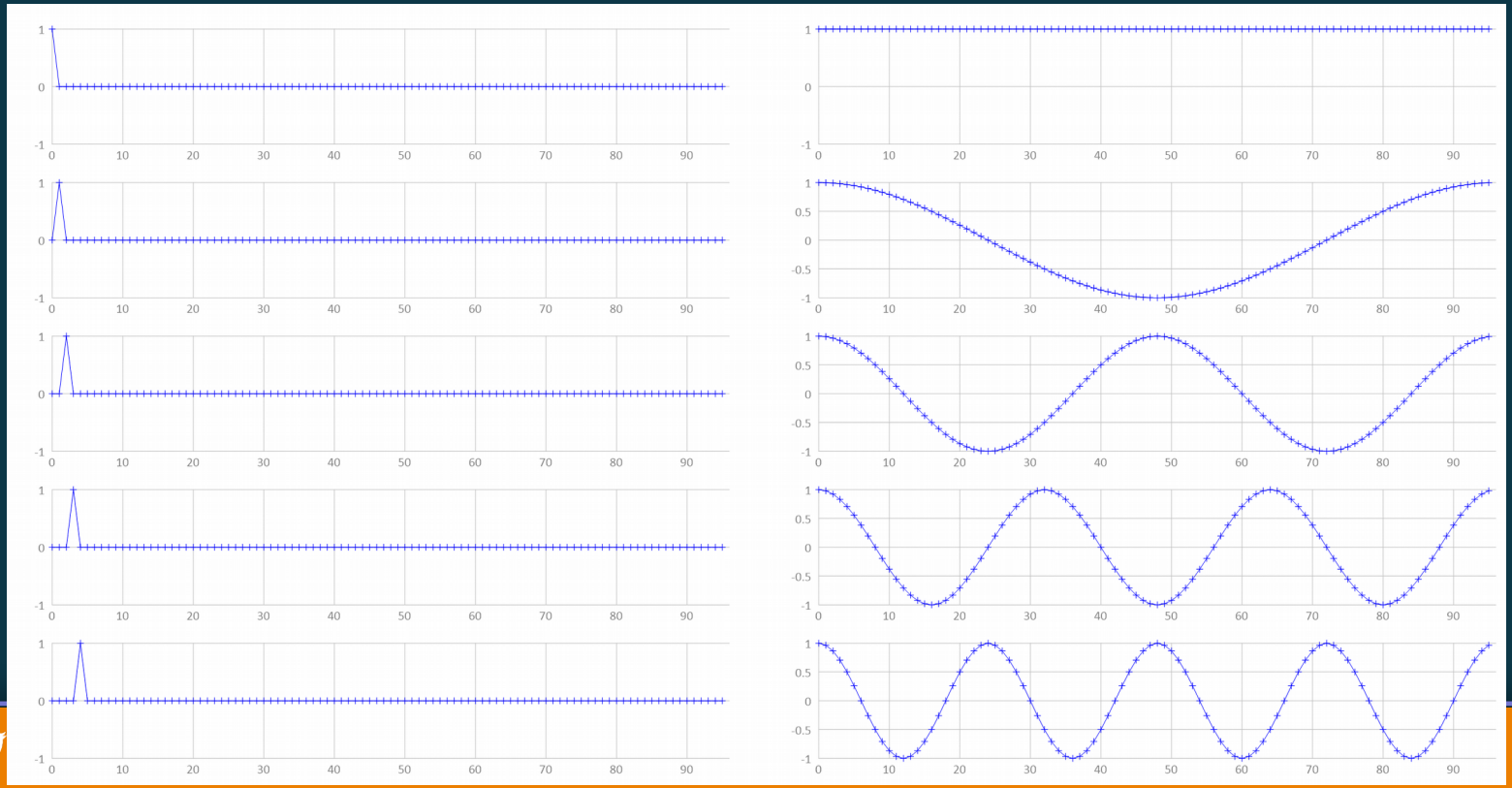
- Domain : time (1d) + frequency (1d)
- Image : magnitude (1d)

- Notion of frequency

Time-wise representation of a sound



Crash course: Fourier transform (FT)

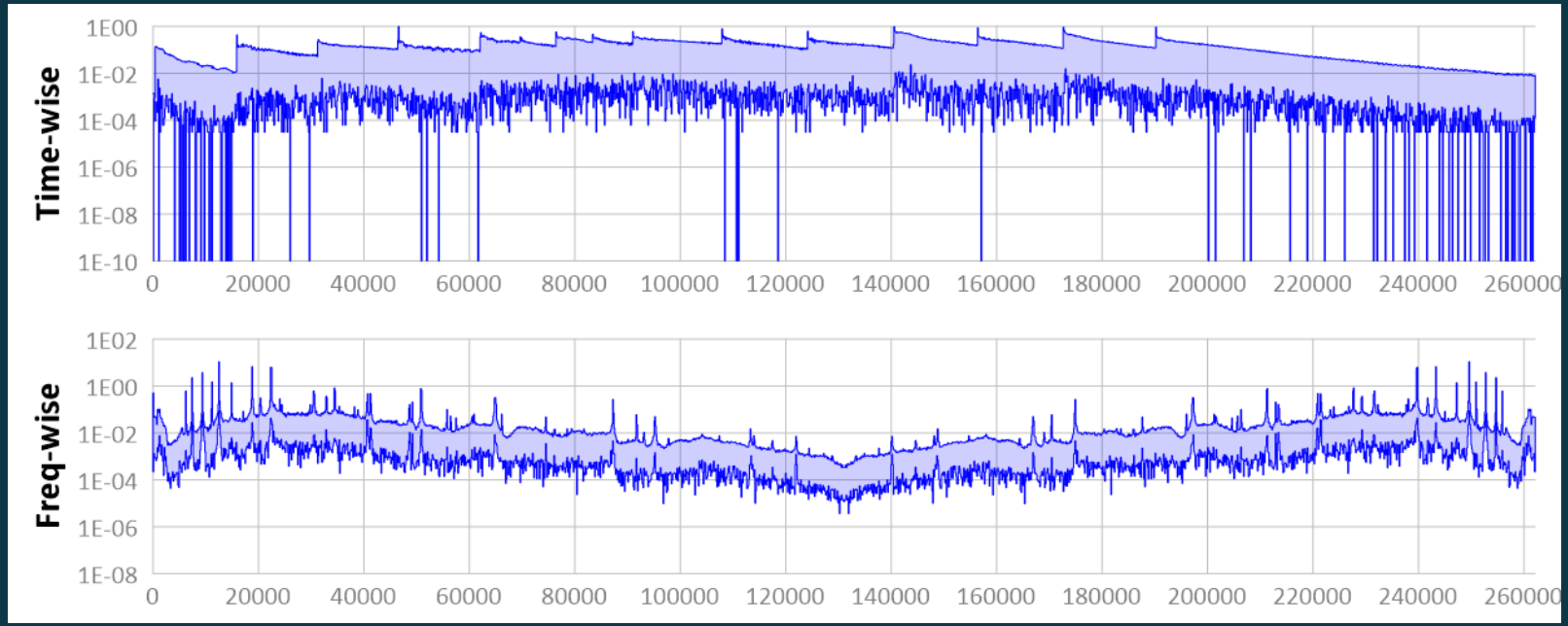


Crash course: Fourier transform (FT)

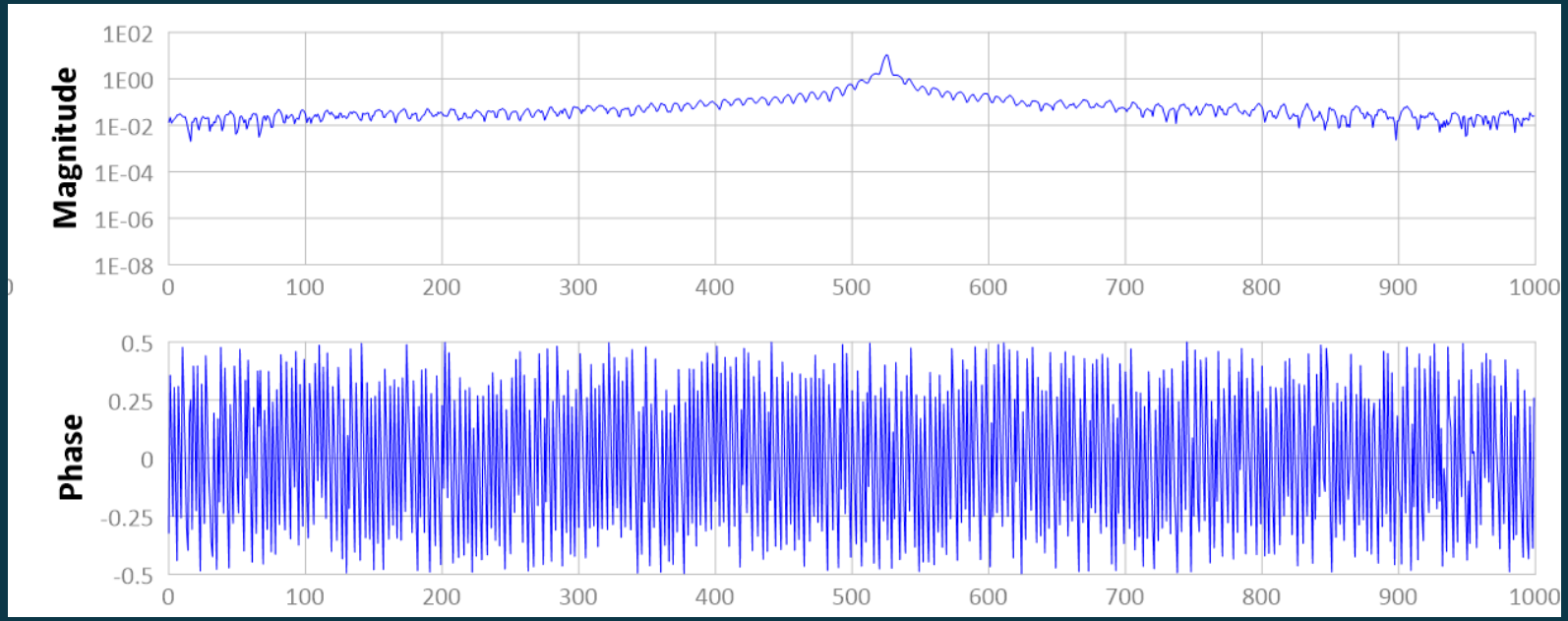
- `output[i] ← atoms[i;] +.× input`
- `output ← atoms +.× input`

- The FT is non-local (sines are non-zero)
- The concept of frequency requires complex scalars (sines have magnitude and phase)
- Atom matrix wraps in both directions

Frequency-wise representation of a sound



Frequency-wise representation of a sound



Frequency-wise representation of a sound

- Domain: time (1d)
- Image : amplitude (1d)

- Domain: frequency (1d)
- Image : magnitude (1d) + phase (1d)

  NB. number unique samples: `⌈1+0.5×≠ω`

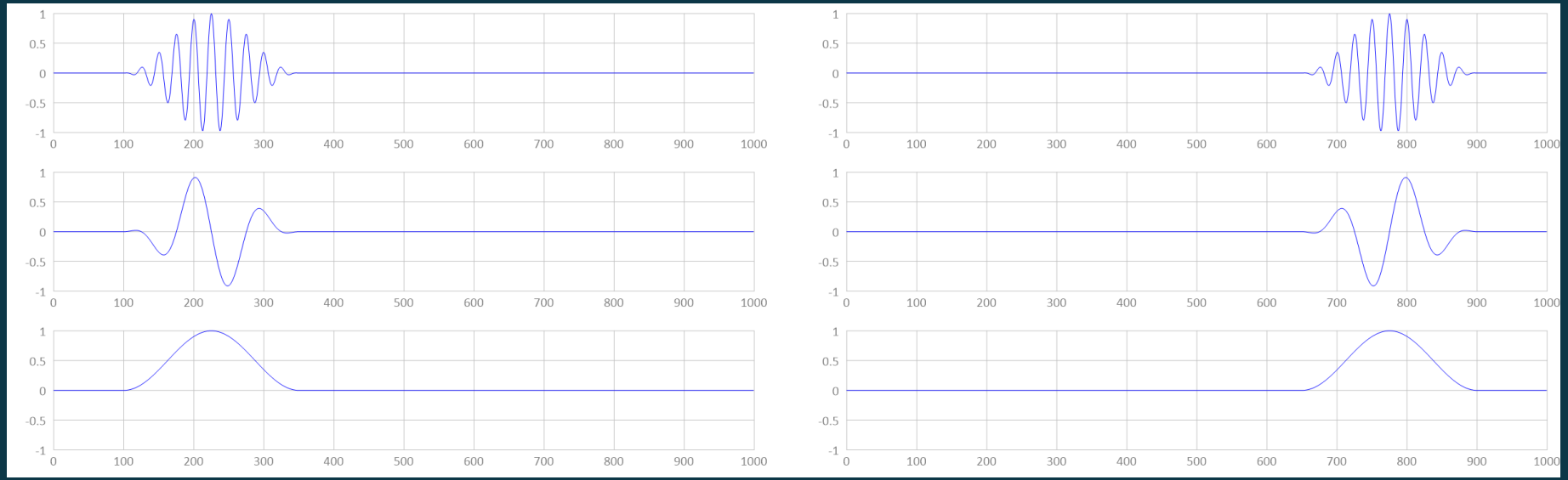
Degrees of freedom of a sound

- Domain : time (1d) + frequency (1d)
- Image : magnitude (1d)

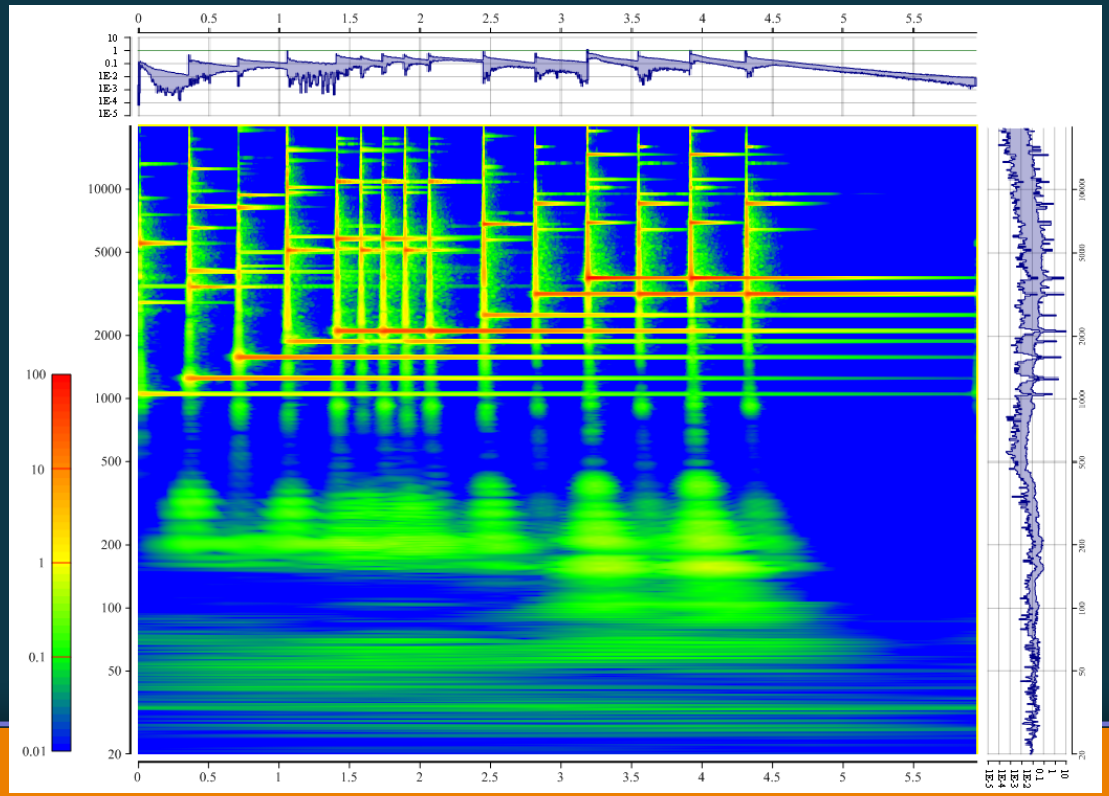
- Domain: frequency (1d)
- Image : magnitude (1d) + phase (1d)

- Notion of time-limited frequency

Crash course: short-time FT (frames)



The time-frequency plane



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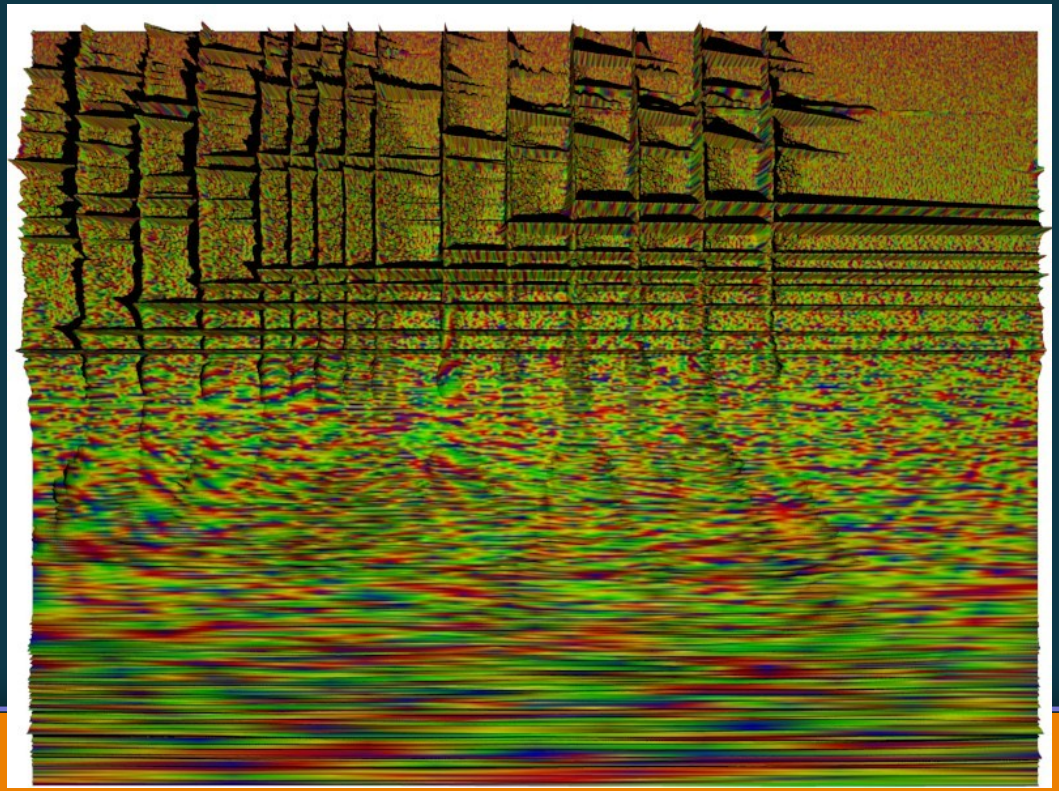
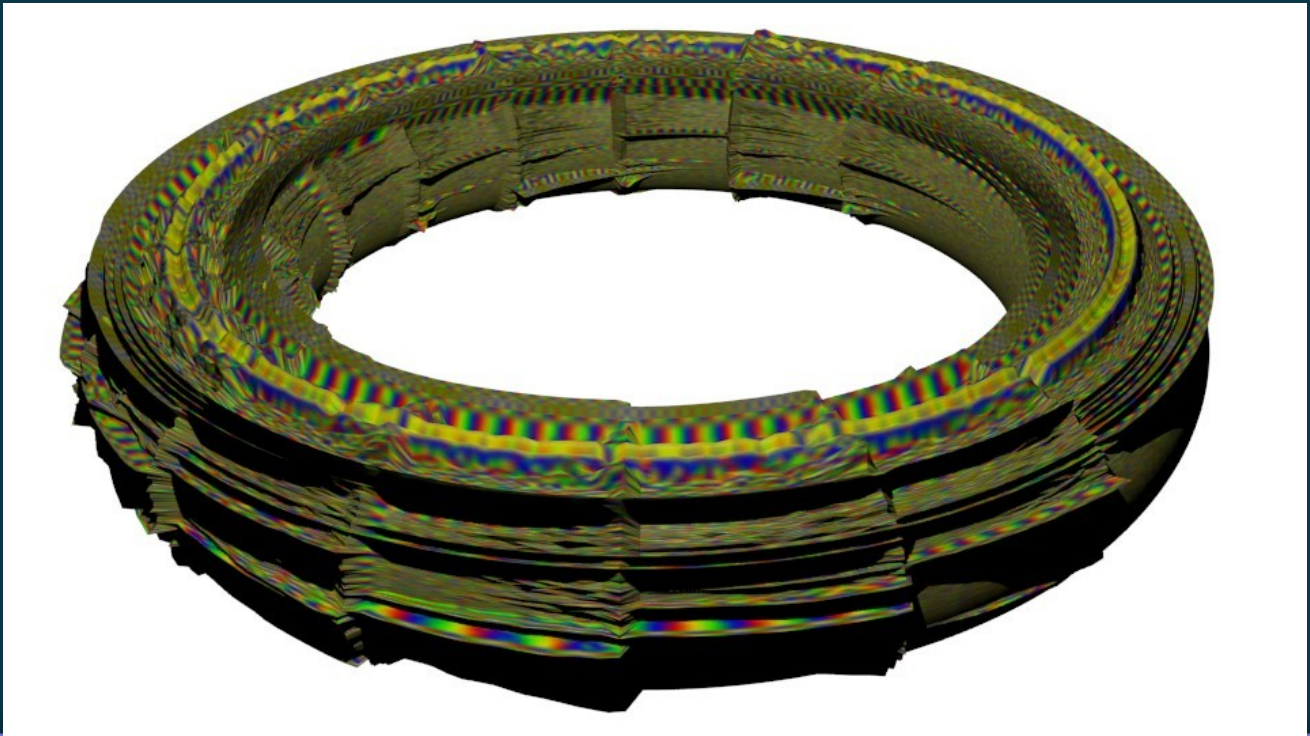


The time-frequency plane



The time-frequency plane

- Domain : time (1d) + frequency (1d)
- Image : magnitude (1d)

- Domain: time (1d) + frequency (1d)
- Image : magnitude (1d) + phase (1d)

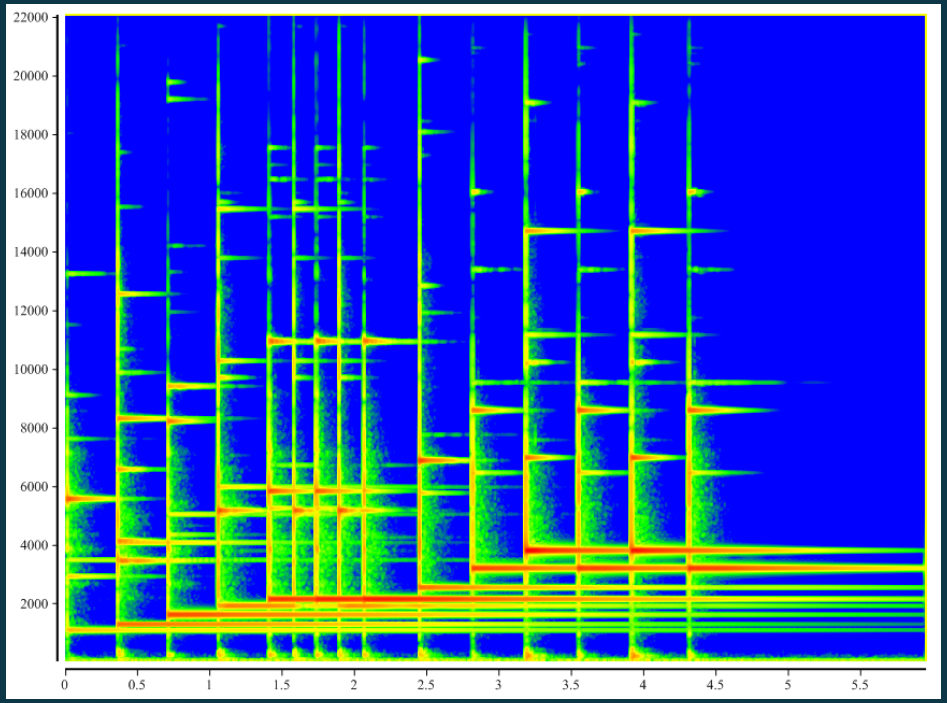
- Happy end?

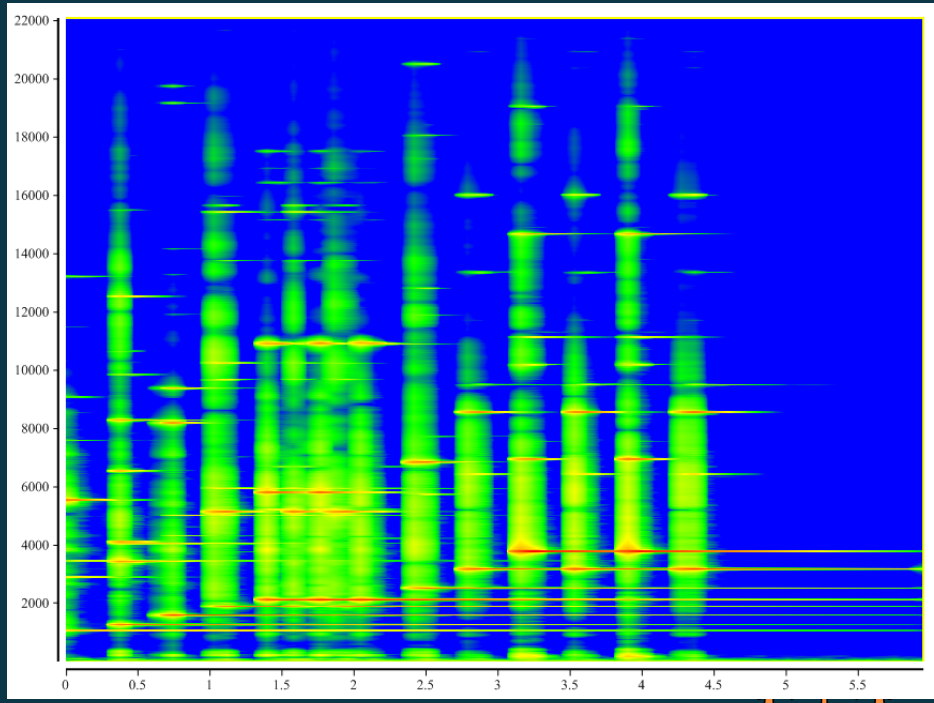
The ~~uncertainty~~ dispersion principle

- Dispersion(time) × Dispersion(frequency) ≥ constant
- Continuous FT: standard deviation (Heisenberg)
- Discrete FT: number of non-zero values (support)

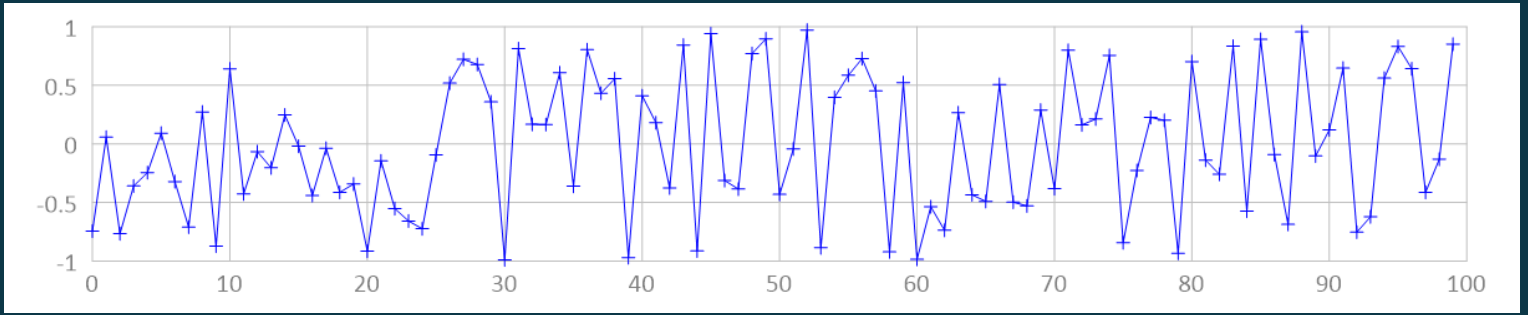
- A signal cannot be time-limited and frequency-limited
- Information in non-local in at least one domain
- Time-frequency information is frame-dependent (subjective)

The ~~uncertainty~~ dispersion principle



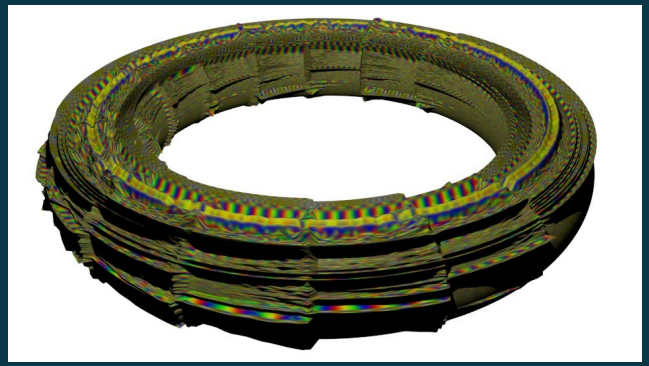


How many dimensions does a vector have?



- Domain: time (1d)
- Image: amplitude (1d)

How many dimensions does a vector have?



- Domain : time (0.5d?) + frequency (0.5d?)
- Image : magnitude (1d) + phase (1d)
- Dual domain is introduced and restricted by a choice
- There are more (invertible?) choices than signals

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What's a dimension anyway?

Nicolas Delcros

Nic's question

- ℚ: (a/b) + (c/d) = (ad+bc) / (bd)
- ℂ: (a+ib) × (c+id) = (ac-bd) + i(ad+bc)

- Why should scalars be 2d?
- Can the brain cope with complex not being 2d?