



Rectangles All The Way Down

Martin Thompson - @mjpt777

“The most amazing achievement of the computer software industry is its continuing cancellation of the steady and staggering gains made by the computer hardware industry.”

- Henry Peteroski

Fundamental Laws

CPU Performance – Memory Lane

◆ ***“Transistor density doubles every year”***

- Gordon Moore

1965



CPU Performance – Memory Lane

◆ ***“Transistor density doubles every 2 years”***

- *Gordon Moore*

◆ ***“Transistor density doubles every year”***

- *Gordon Moore*

1965

1975



CPU Performance – Memory Lane

◆ ***“CPUs double in speed every 18 months”***

- *David House*

◆ ***“Transistor density doubles every 2 years”***

- *Gordon Moore*

◆ ***“Transistor density doubles every year”***

- *Gordon Moore*

1965

1975



CPU Performance – Memory Lane

“The free lunch is over:” ◆
- Herb Sutter

◆ ***“CPUs double in speed every 18 months”***
- David House

◆ ***“Transistor density doubles every 2 years”***
- Gordon Moore

◆ ***“Transistor density doubles every year”***
- Gordon Moore



CPU Performance – Memory Lane

Retirement of Tick Tock ◆

- Intel

“The free lunch is over:” ◆

- Herb Sutter

◆ **“CPUs double in speed every 18 months”**

- David House

◆ **“Transistor density doubles every 2 years”**

- Gordon Moore

◆ **“Transistor density doubles every year”**

- Gordon Moore



CPU Performance – Memory Lane

Spectre & Meltdown ◆

- Google

Retirement of Tick Tock ◆

- Intel

“The free lunch is over:” ◆

- Herb Sutter

◆ **“CPUs double in speed every 18 months”**

- David House

◆ **“Transistor density doubles every 2 years”**

- Gordon Moore

◆ **“Transistor density doubles every year”**

- Gordon Moore



Concurrency & Parallelism



Universal Scalability Law (USL)

$$C(N) = N / (1 + \alpha(N - 1) + ((\beta * N) * (N - 1)))$$

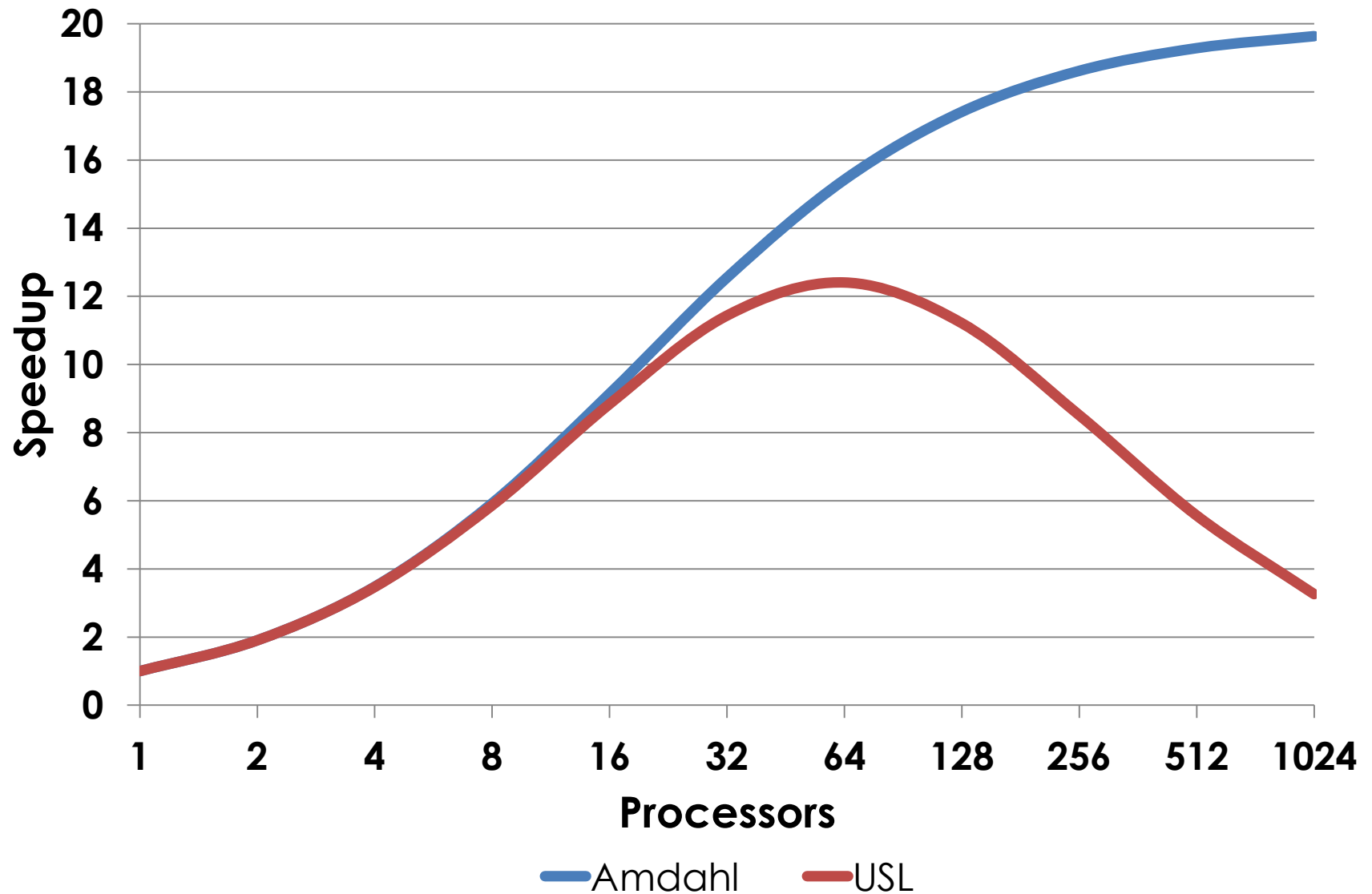
C = capacity or throughput

N = number of processors

α = **contention** penalty

β = **coherence** penalty

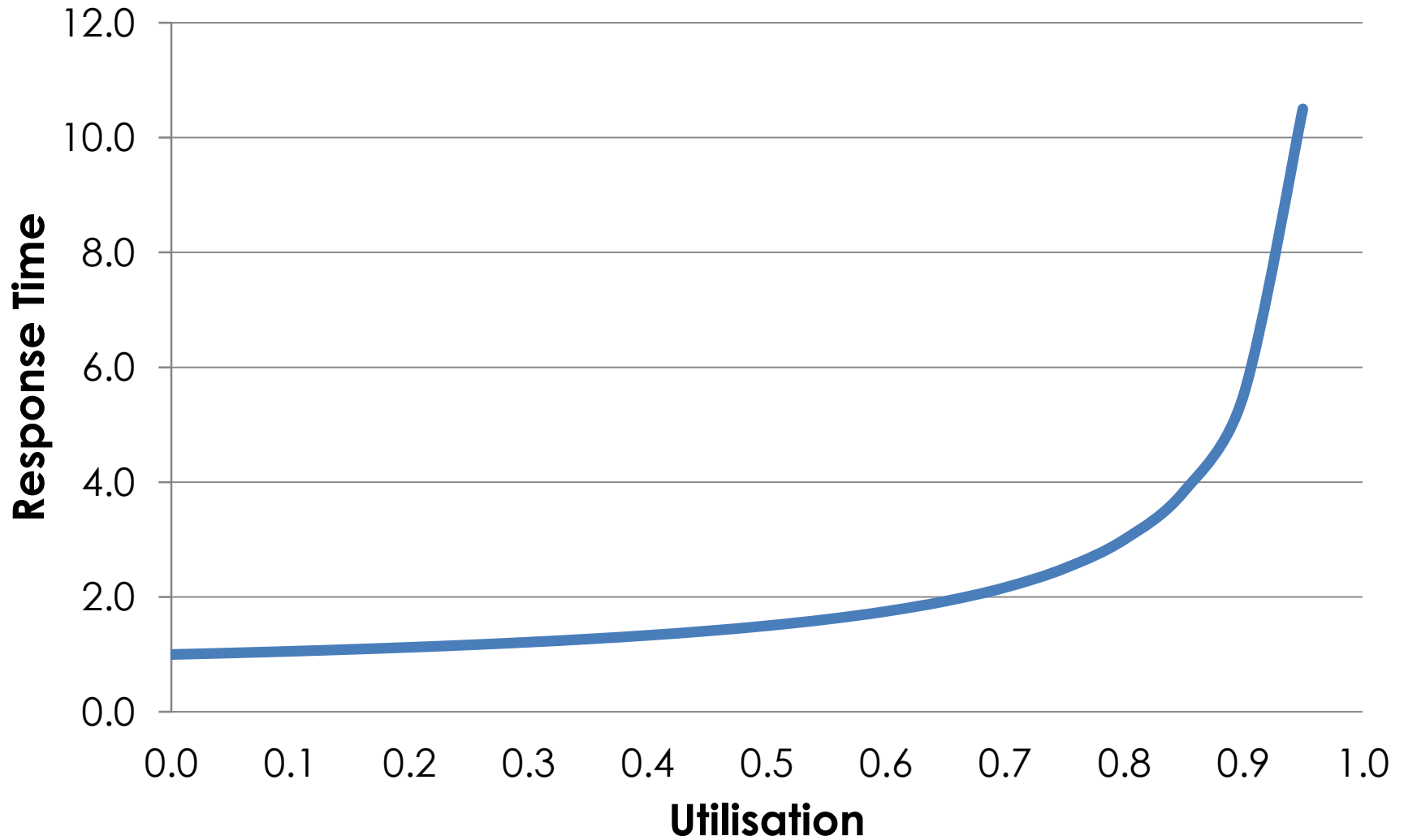
Universal Scalability Law (USL)



**If concurrency is so difficult then
what else can we do?**



Queueing Theory



Queueing Theory

$$r = s (2 - \rho) / 2 (1 - \rho)$$

r = mean response time

s = service time

ρ = utilisation

Note: $\rho = \lambda * s$



Little's Law

$$L = \lambda W$$

$$\text{WIP} = \text{Throughput} * \text{Cycle Time}$$

Little's Law

$$L = \lambda W$$

$$\text{WIP} = \text{Throughput} * \text{Cycle Time}$$

Bandwidth Delay Product:

$$\text{Bytes in flight} = \text{Bandwidth} * \text{Latency}$$

Little's Law

$$L = \lambda W$$

$$\text{WIP} = \text{Throughput} * \text{Cycle Time}$$

Bandwidth Delay Product:

$$\text{Bytes in flight} = \text{Bandwidth} * \text{Latency}$$

$$80 \text{ bytes} / 100\text{ns} = 800 \text{ MB/s} : 10 \text{ LFBs}$$

Memory

**Are all memory
operations equal?**

Sequential Access

-

**Average time in ns/op to sum all
longs in a 1GB array?**

Access Pattern Benchmark

Benchmark	Score	Error	Units
=====	=====	=====	=====
sequential	0.832	± 0.006	ns/op

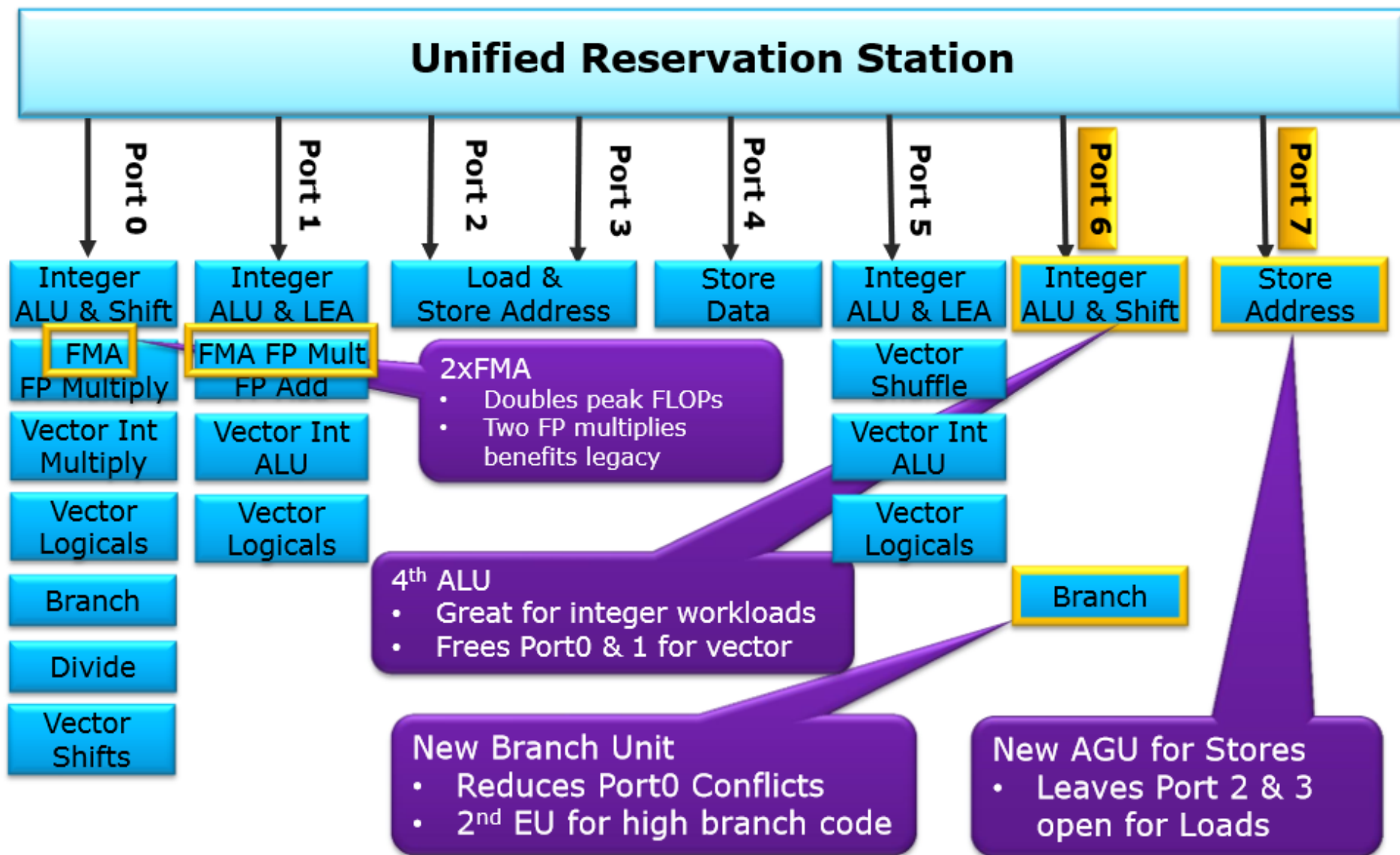
~1 ns/op

Really???

Less than 1ns per operation?

Instruction Level Parallelism

Haswell Execution Unit Overview



Access Pattern Benchmark

Benchmark	Score	Error	Units
=====			
sequential	0.832	± 0.006	ns/op
randomPage	2.703	± 0.025	ns/op

Access Pattern Benchmark

Benchmark	Score	Error	Units
=====			
sequential	0.832	\pm 0.006	ns/op
randomPage	2.703	\pm 0.025	ns/op
dependentRandomPage	7.102	\pm 0.326	ns/op

Access Pattern Benchmark

Benchmark	Score	Error	Units
=====			
sequential	0.832	\pm 0.006	ns/op
randomPage	2.703	\pm 0.025	ns/op
dependentRandomPage	7.102	\pm 0.326	ns/op
randomHeap	19.896	\pm 3.110	ns/op

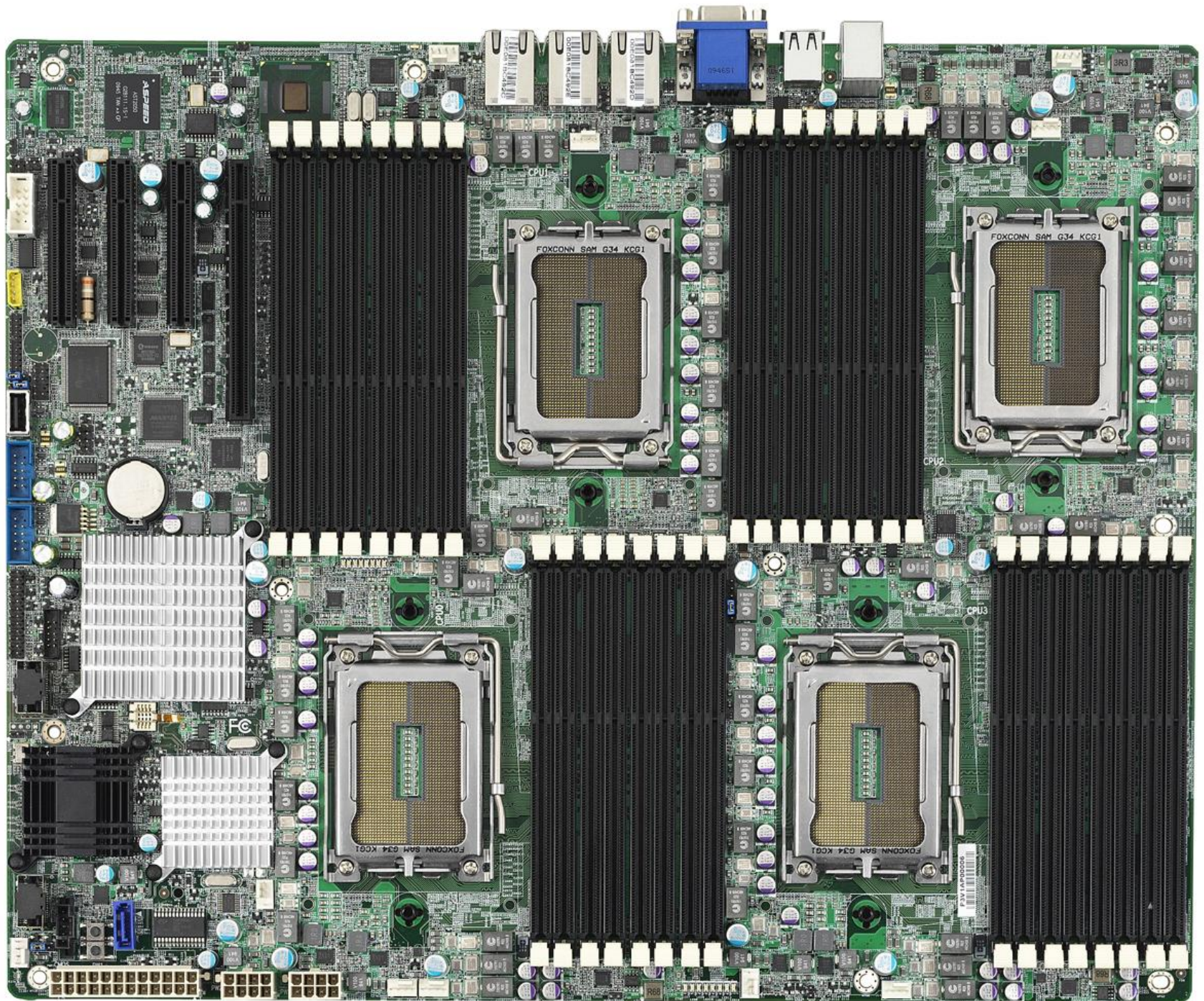
Access Pattern Benchmark

Benchmark	Score	Error	Units
=====			
sequential	0.832	\pm 0.006	ns/op
randomPage	2.703	\pm 0.025	ns/op
dependentRandomPage	7.102	\pm 0.326	ns/op
randomHeap	19.896	\pm 3.110	ns/op
dependentRandomHeap	89.516	\pm 4.573	ns/op

Access Pattern Benchmark

Benchmark	Score	Error	Units
=====			
sequential	0.832	± 0.006	ns/op
randomPage	2.703	± 0.025	ns/op
dependentRandomPage	7.102	± 0.326	ns/op
randomHeap	19.896	± 3.110	ns/op
dependentRandomHeap	89.516	± 4.573	ns/op

~90 ns/op



**A 100ns cache-miss is a
lost opportunity to execute
~1000 instructions on CPU**

Algorithms & Data Structures

Little's Law

$$L = \lambda W$$

Bandwidth Delay Product:

Bytes in flight = Bandwidth * Latency

80 bytes / 100ns = 800 MB/s :10 LFBs

Little's Law

$$L = \lambda W$$

Bandwidth Delay Product:

Bytes in flight = Bandwidth * Latency

80 bytes / 100ns = 800 MB/s :10 LFBs

80 bytes / 15ns = 5.3 GB/s :prefetch

Little's Law

$$L = \lambda W$$

Bandwidth Delay Product:

Bytes in flight = Bandwidth * Latency

80 bytes / 100ns = 800 MB/s :10 LFBs

80 bytes / 15ns = 5.3 GB/s :prefetch

640 bytes / 15ns = 42.6 GB/s :cachelines

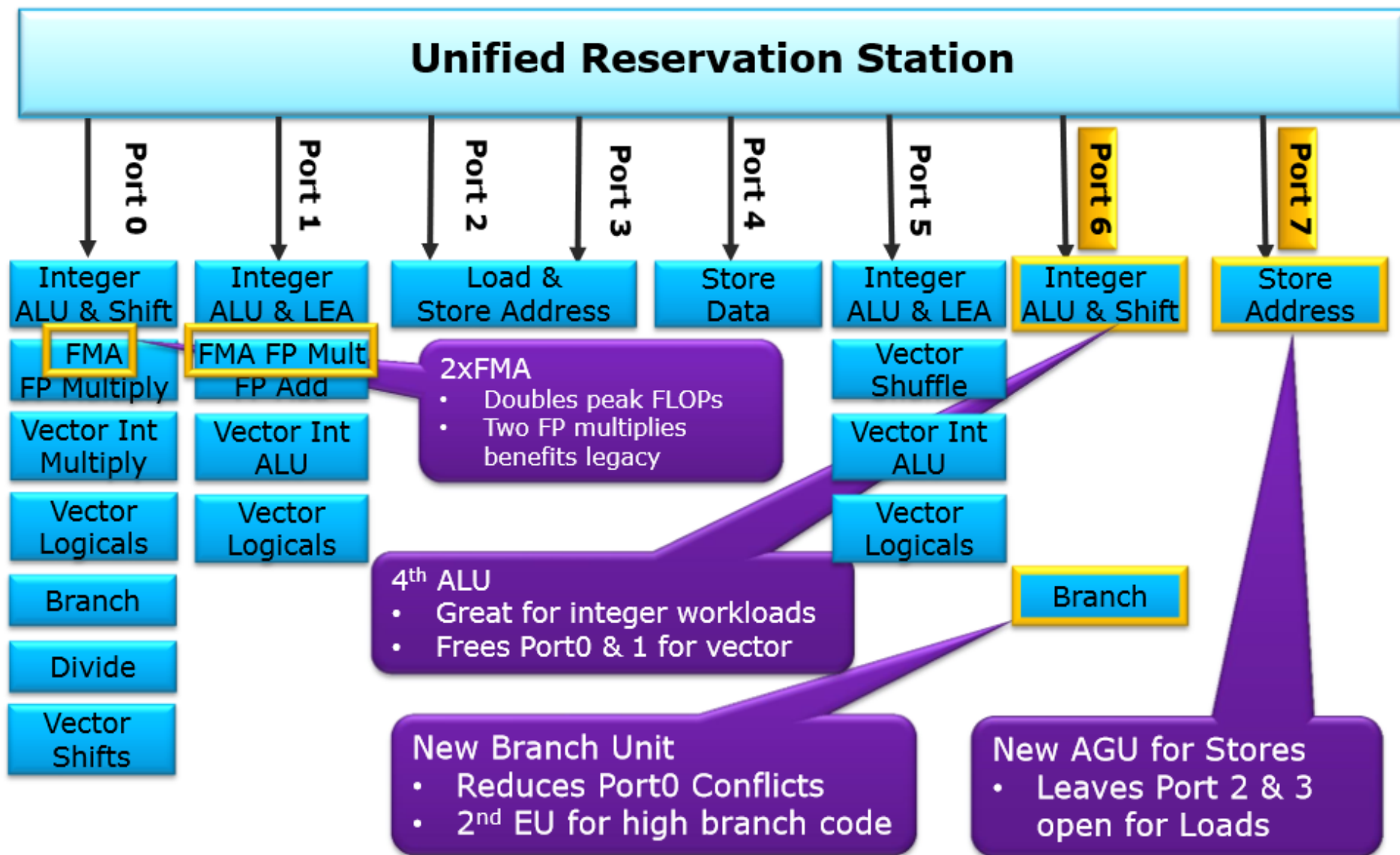
***Arrays are the most efficient
data structure to traverse***



***Functional data structures
are like sausages,
the more you see them being
made, the less well you will sleep***

Branches

Haswell Execution Unit Overview



Branch Benchmark

Benchmark	Score		Error	Units
=====				
baseline	585.600	±	4.469	us/op

Branch Benchmark

Benchmark	Score		Error	Units
=====				
baseline	585.600	±	4.469	us/op
predictable	578.364	±	10.906	us/op

Branch Benchmark

Benchmark	Score		Error	Units
=====				
baseline	585.600	±	4.469	us/op
predictable	578.364	±	10.906	us/op
unPredictable	2234.414	±	564.472	us/op

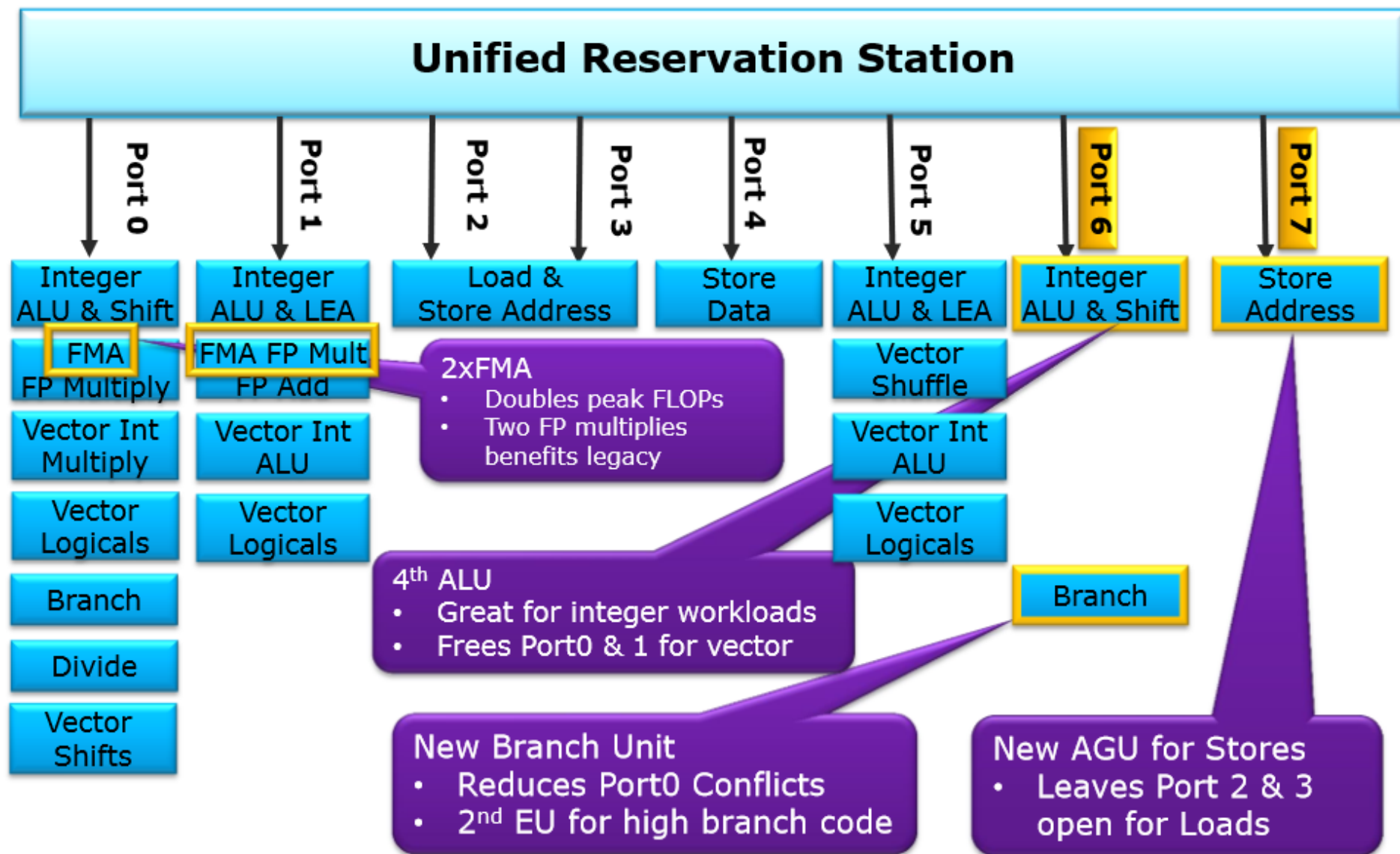
What can we do?

Count bits as Booleans

Wide Registers

Math, Data Dependencies, and Instruction Level Parallelism

Haswell Execution Unit Overview



Consider Sorting Arrays

Programming
Techniques

G. Manacher
Editor

Multiple Byte Processing with Full- Word Instructions

Leslie Lamport
Massachusetts Computer Associates, Inc.

A method is described which allows parallel processing of packed data items using only ordinary full-word computer instructions, even though the processing requires operations whose execution is contingent upon the value of a datum. It provides a useful technique for processing small data items such as alphanumeric characters.

Key Words and Phrases: byte processing, character processing, packed data

CR Categories: 4.9

Communications
of
the ACM

August 1975
Volume 18
Number 8

“It’s a neat hack, and it’s more useful now than it was then for two reasons.”

- Leslie Lamport (2011)

“The obvious reason is that word size is larger now, with many computers having 64-bit words.”

- Leslie Lamport (2011)

“The less obvious reason is that conditional operations are implemented with masking rather than branching.”

- Leslie Lamport (2011)

“Branching is more costly on modern multi-issue computers than it was on the computers of the 70s.”

- Leslie Lamport (2011)

Programming
Techniques

S.L. Graham, R.L. Rivest
Editors

Counting Large Numbers of Events in Small Registers

Robert Morris
Bell Laboratories, Murray Hill, N.J.

It is possible to use a small counter to keep approximate counts of large numbers. The resulting expected error can be rather precisely controlled. An example is given in which 8-bit counters (bytes) are used to keep track of as many as 130,000 events with a relative error which is substantially independent of the number n of events. This relative error can be expected to be 24 percent or less 95 percent of the time (i.e. $\sigma = n/8$). The techniques could be used to advantage in multichannel counting hardware or software used for the monitoring of experiments or processes.

Key Words and Phrases: counting

CR Categories: 5.11

Work with your CPU caches

Memory Access Considerations

1. **Temporal:** group accesses in time

Memory Access Considerations

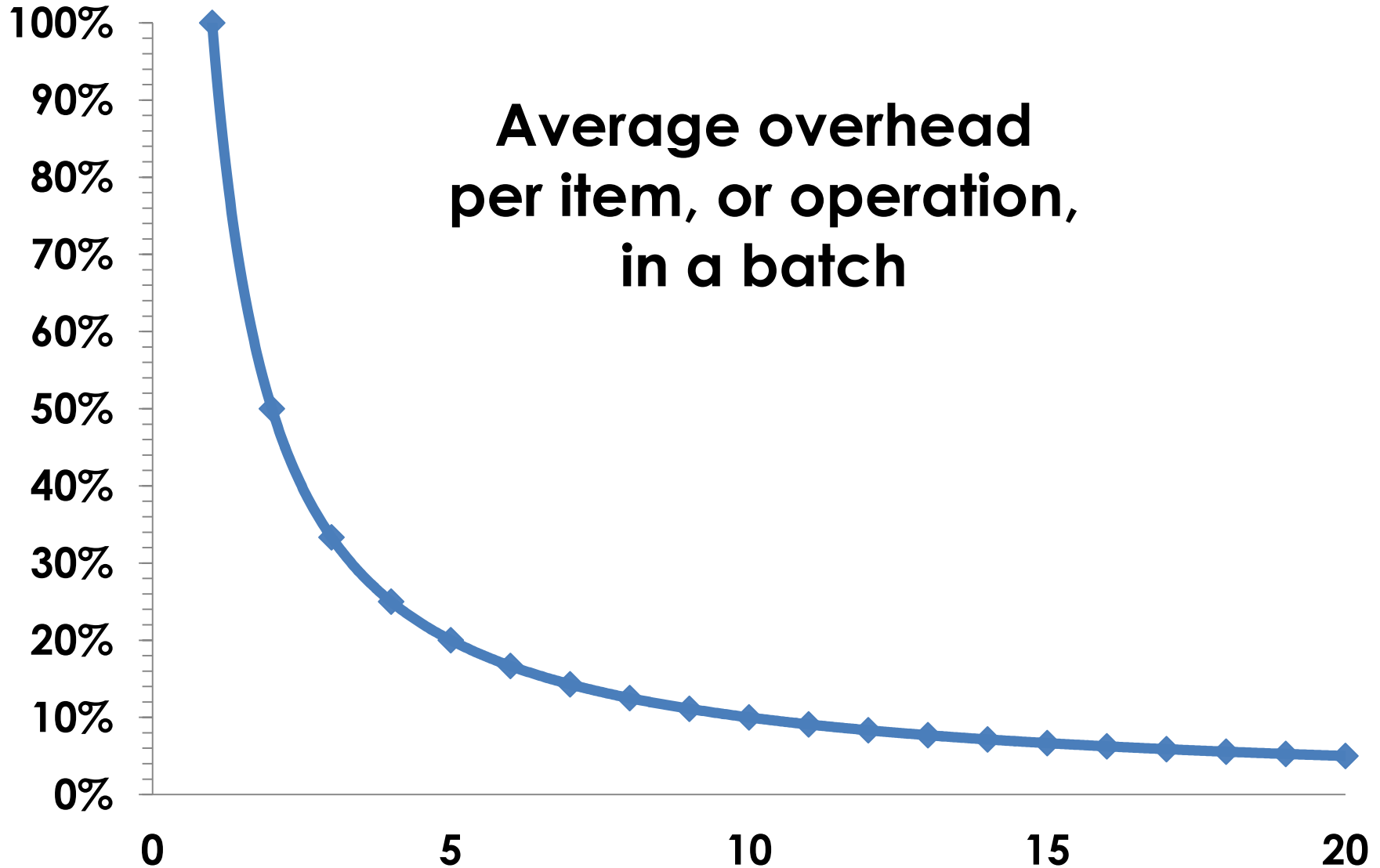
1. **Temporal:** group accesses in time
2. **Spatial:** group access in space

Memory Access Considerations

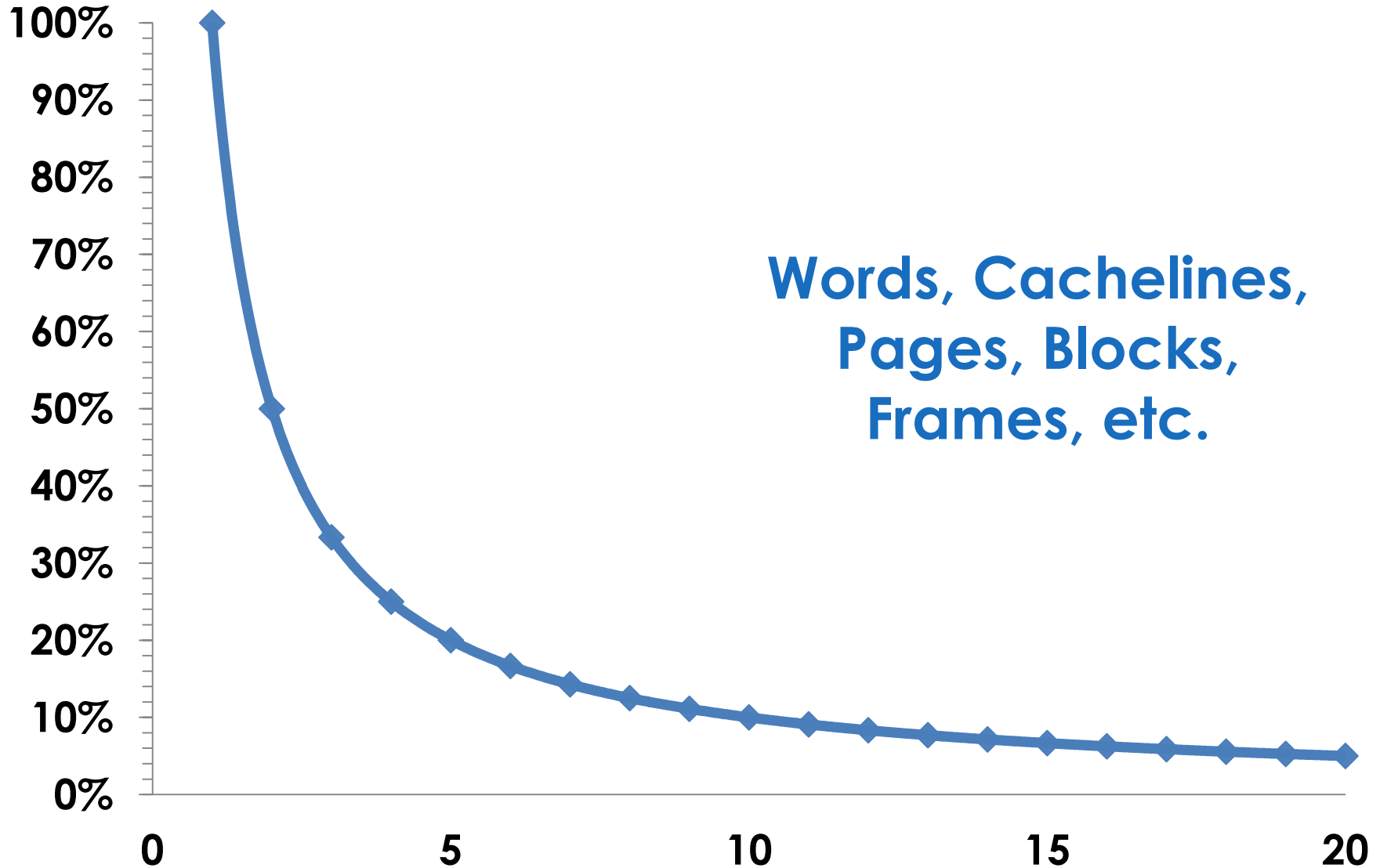
1. **Temporal:** group accesses in time
2. **Spatial:** group access in space
3. **Pattern:** create predictable patterns

Batching

Batching – Amortising Costs



Batching – Amortising Costs



In closing...

Profile, profile, profile...

Eliminate Waste
Batch to Amortise
Access Memory in Patterns
Favour Math over Branches
Favour Predictable Branches

Consider Parallelism

-

ILP & Task

Is it really “Turtles all the way down”?



Rectangles all the way down...

Is it really “Turtles all the way down”?

- Networks: **Frames**
- Operating Systems: **Pages**
- File systems and storage: **Blocks**
- DRAM memory: **Banks** and **Row Buffers**
- CPU cache subsystems: **Cache Lines**
- Applications use **Arrays** plus and interesting data structures are made up of small **Arrays**



“I don’t care what data structure you use, nothing beats an array”

- a HFT Programmer

Questions?

Twitter: @mjpt777

“Travel is fatal to prejudice, bigotry, and narrow-mindedness, and many of our people need it sorely on these accounts. Broad, wholesome, charitable views of men and things cannot be acquired by vegetating in one little corner of the earth all one's lifetime.”

- Mark Twain